Self-similar growth-fragmentations as scaling limits of Markov branching processes

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Les probabilités de demain
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1 Motivating example

2 Self-similar growth-fragmentation

3 Markov branching process

4 Scaling limits

5 Two difficulties induced by growth

Results

6 Assumptions

7 Scaling limit for the process

8 Scaling limit for the tree

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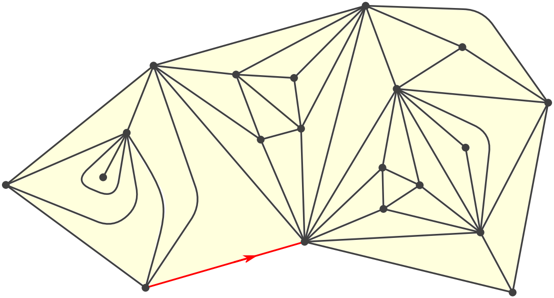
Motivating example

Peeling a random Boltzmann triangulation of the n-gon



Motivating example

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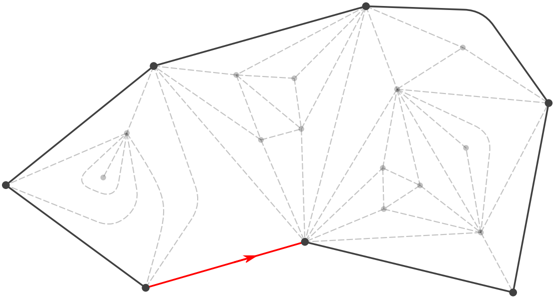

Hole perimeters:

$$\boldsymbol{X}^{(n)}(0) = (7, 0, \dots)$$

Motivating example

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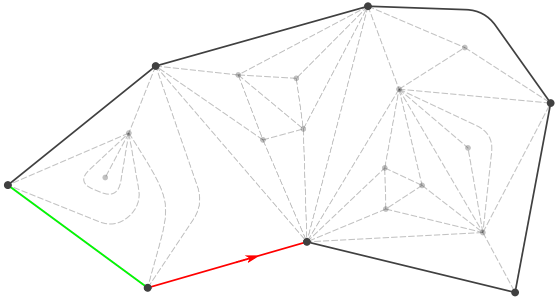

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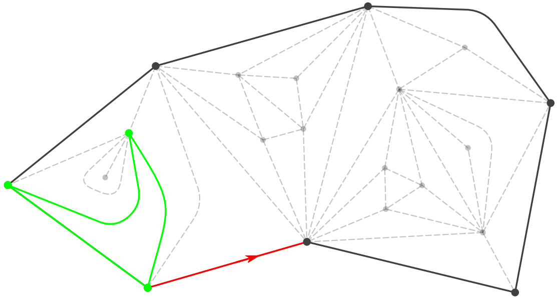

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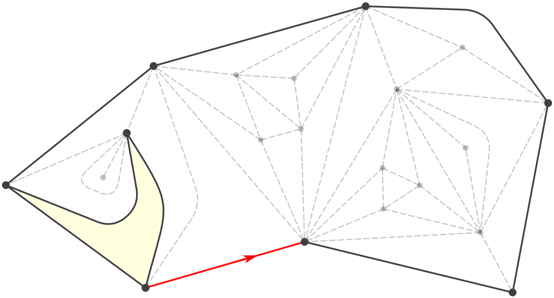
Motivating example

Peeling a random Boltzmann triangulation of the n-gon

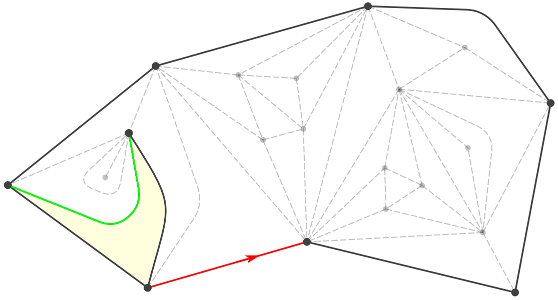


Hole perimeters:

$$\boldsymbol{X}^{(n)}(1) = (8, 0, \dots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon



Hole perimeters:

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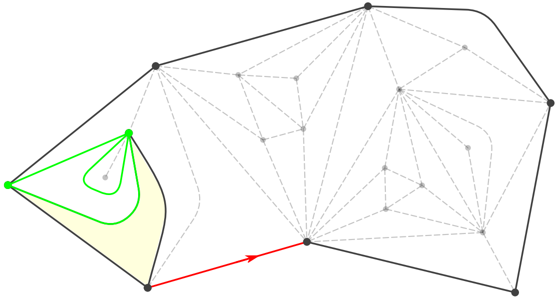
Motivating example

Peeling a random Boltzmann triangulation of the n-gon

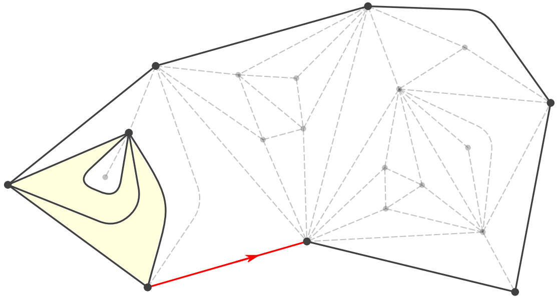


Hole perimeters:

$$\boldsymbol{X}^{(n)}(1) = (8, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

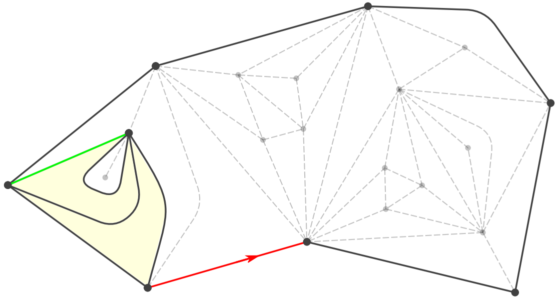


Hole perimeters:

$$\boldsymbol{X}^{(n)}(2) = (8, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

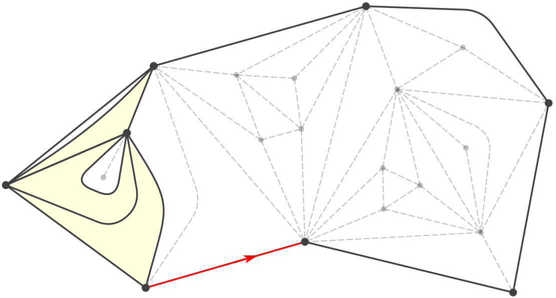


Hole perimeters:

$$\boldsymbol{X}^{(n)}(2) = (8, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

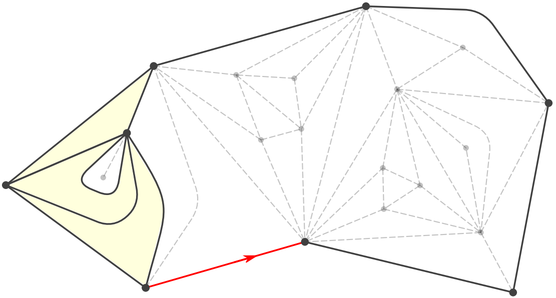


Hole perimeters:

$$\boldsymbol{X}^{(n)}(3) = (7, 2, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

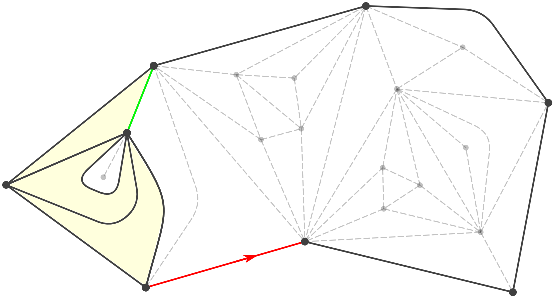


Hole perimeters:

$$\boldsymbol{X}^{(n)}(4) = (7, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

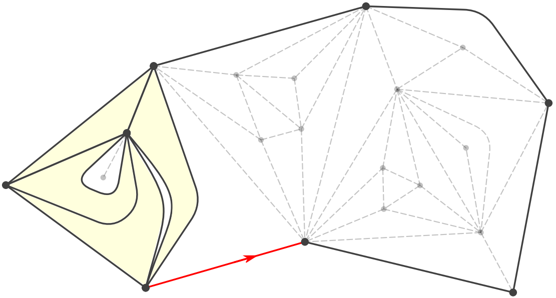


Hole perimeters:

$$\boldsymbol{X}^{(n)}(4) = (7, 1, 0, \dots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

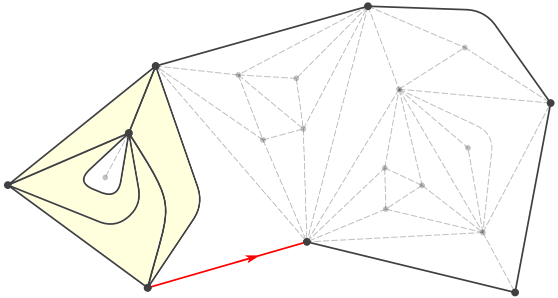


Hole perimeters:

$$\boldsymbol{X}^{(n)}(5) = (6, 2, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

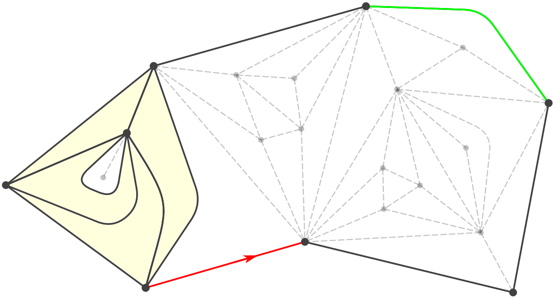


Hole perimeters:

$$\boldsymbol{X}^{(n)}(6) = (6, 1, 0, \dots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

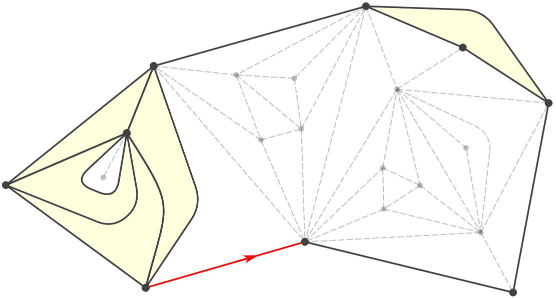


Hole perimeters:

$$\boldsymbol{X}^{(n)}(6) = (6, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

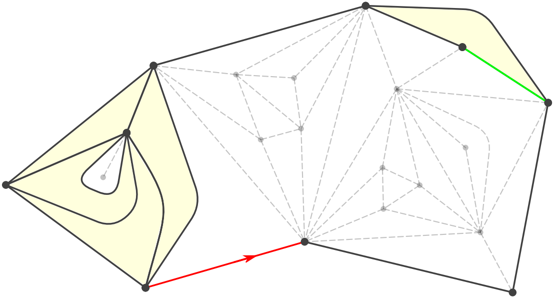


Hole perimeters:

$$\boldsymbol{X}^{(n)}(7) = (7, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

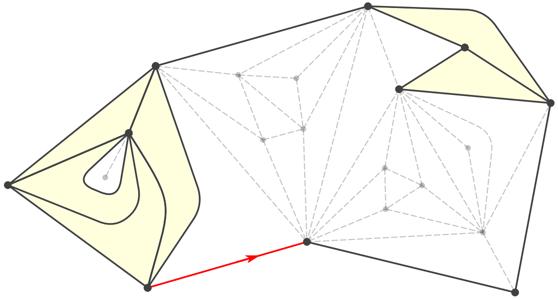


Hole perimeters:

$$\boldsymbol{X}^{(n)}(7) = (7, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

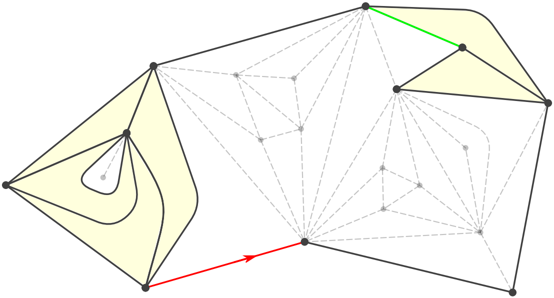


Hole perimeters:

$$\boldsymbol{X}^{(n)}(8) = (8, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

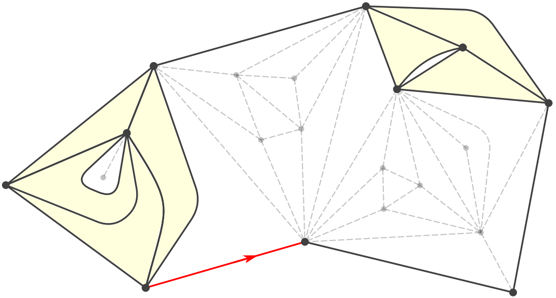


Hole perimeters:

$$\boldsymbol{X}^{(n)}(8) = (8, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

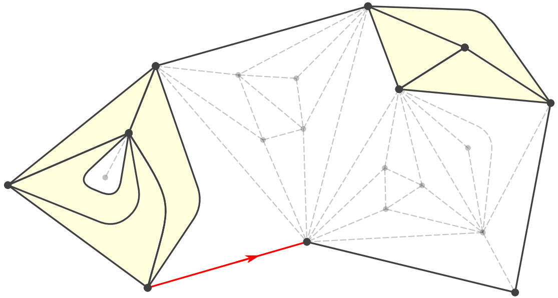


Hole perimeters:

$$\boldsymbol{X}^{(n)}(9) = (7, 2, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

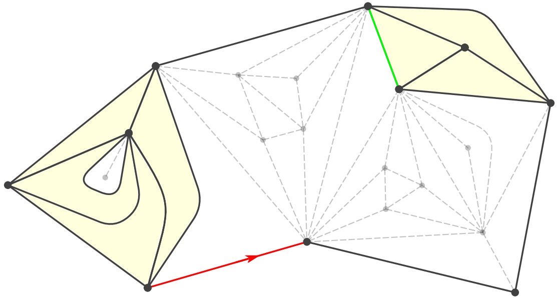


Hole perimeters:

$$\boldsymbol{X}^{(n)}(10) = (7, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

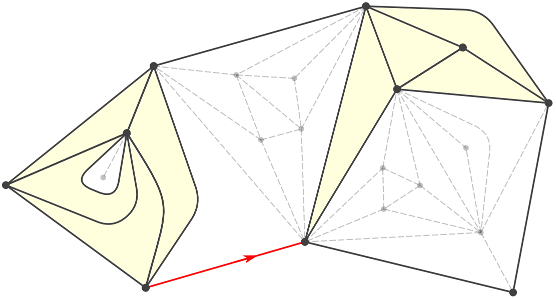


Hole perimeters:

$$\boldsymbol{X}^{(n)}(10) = (7, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

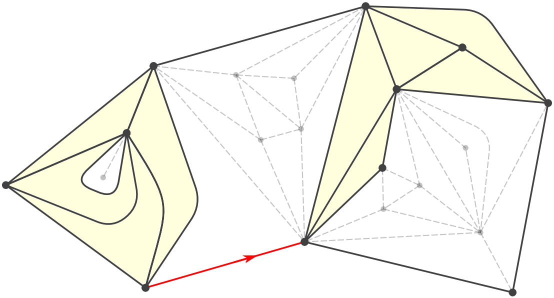


Hole perimeters:

$$\boldsymbol{X}^{(n)}(11) = (4, 4, 1, 0, \dots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

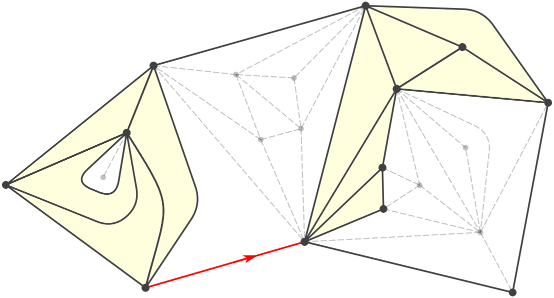


Hole perimeters:

$$\boldsymbol{X}^{(n)}(12) = (5, 4, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

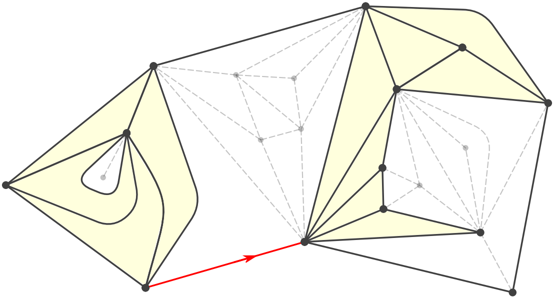


Hole perimeters:

$$\boldsymbol{X}^{(n)}(13) = (6, 4, 1, 0, \dots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

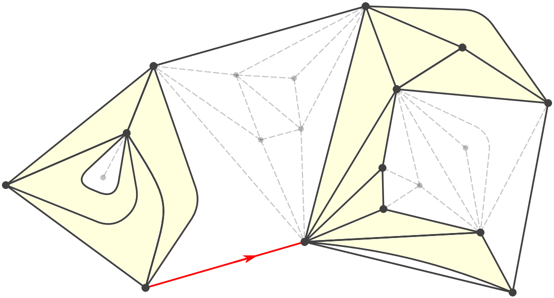


Hole perimeters:

$$\boldsymbol{X}^{(n)}(14) = (7, 4, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

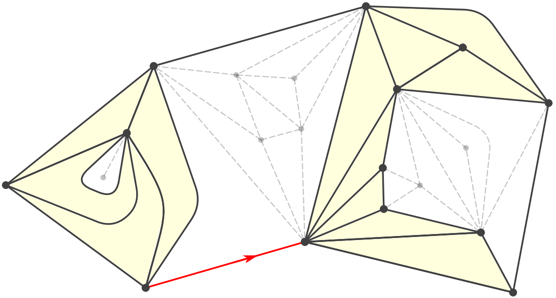


Hole perimeters:

$$\boldsymbol{X}^{(n)}(15) = (6, 4, 2, 1, 0, \dots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

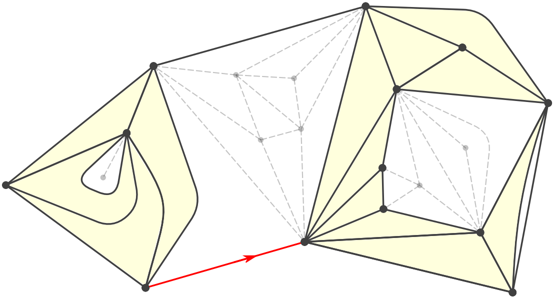


Hole perimeters:

$$\boldsymbol{X}^{(n)}(16) = (6, 4, 1, 0, \dots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

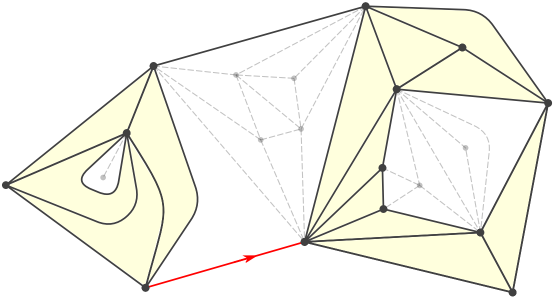


Hole perimeters:

$$\boldsymbol{X}^{(n)}(17) = (5, 4, 2, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

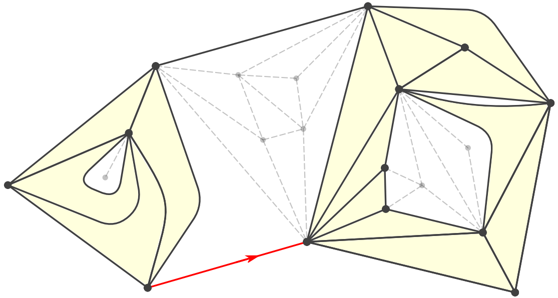


Hole perimeters:

$$\boldsymbol{X}^{(n)}(18) = (5, 4, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

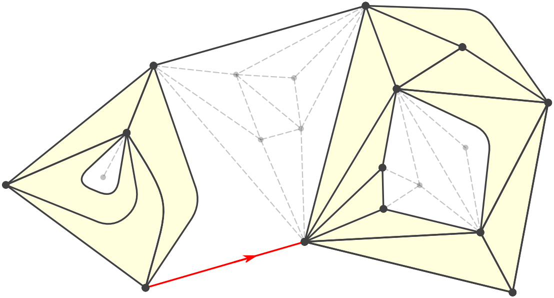


Hole perimeters:

$$\boldsymbol{X}^{(n)}(19) = (4, 4, 2, 1, 0, \dots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

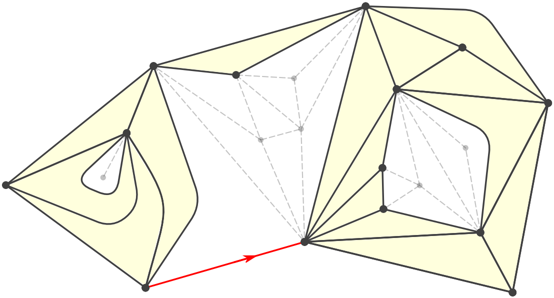


Hole perimeters:

$$\boldsymbol{X}^{(n)}(20) = (4, 4, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

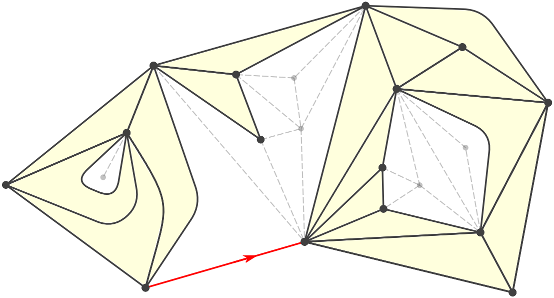


Hole perimeters:

$$\boldsymbol{X}^{(n)}(21) = (5, 4, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

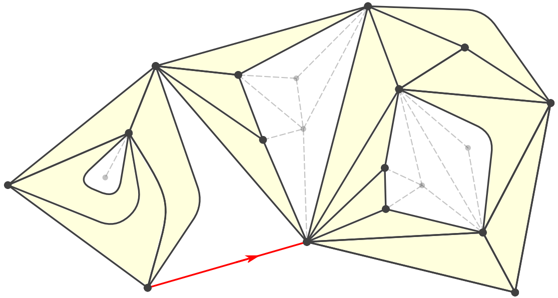


Hole perimeters:

$$\boldsymbol{X}^{(n)}(22) = (6, 4, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

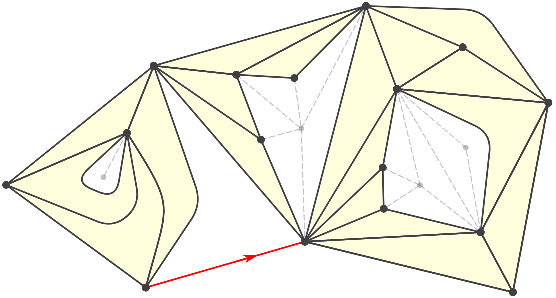


Hole perimeters:

$$\boldsymbol{X}^{(n)}(23) = (4, 4, 3, 1, 0, \dots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

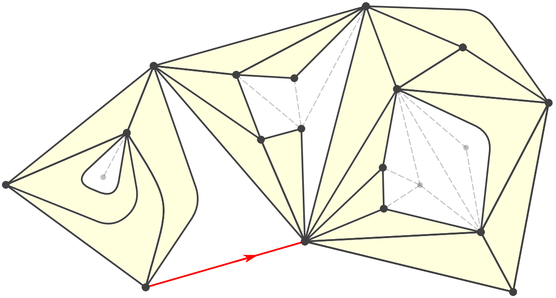


Hole perimeters:

$$\boldsymbol{X}^{(n)}(24) = (5, 4, 3, 1, 0, \dots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

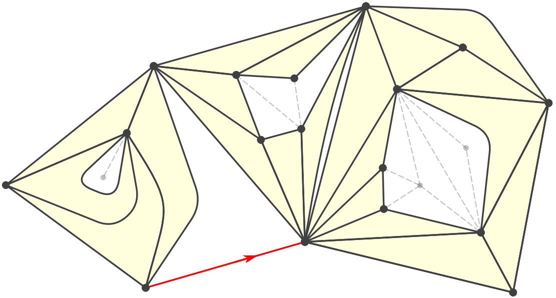


Hole perimeters:

$$\boldsymbol{X}^{(n)}(25) = (6, 4, 3, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

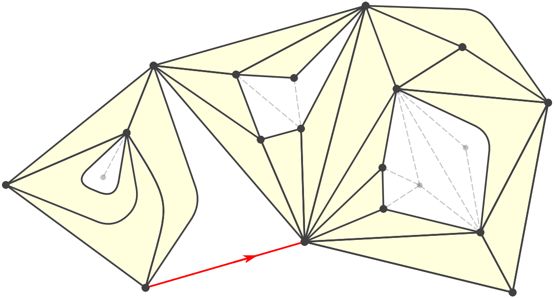


Hole perimeters:

$$\boldsymbol{X}^{(n)}(26) = (5, 4, 3, 2, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

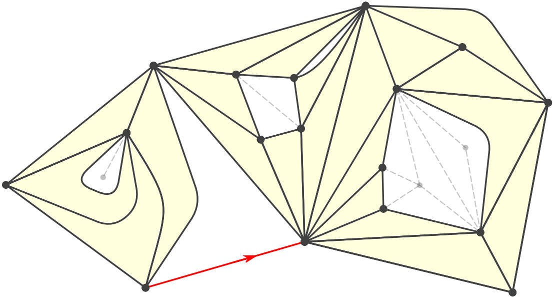


Hole perimeters:

$$\boldsymbol{X}^{(n)}(27) = (5, 4, 3, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

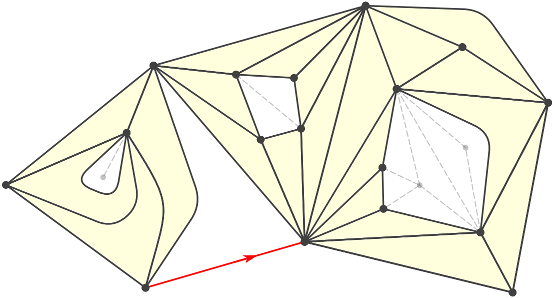


Hole perimeters:

$$\boldsymbol{X}^{(n)}(28) = (4, 4, 3, 2, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

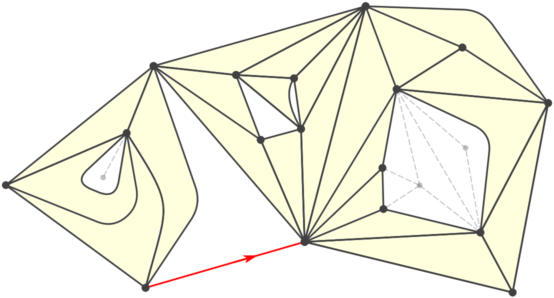


Hole perimeters:

$$\boldsymbol{X}^{(n)}(29) = (4, 4, 3, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

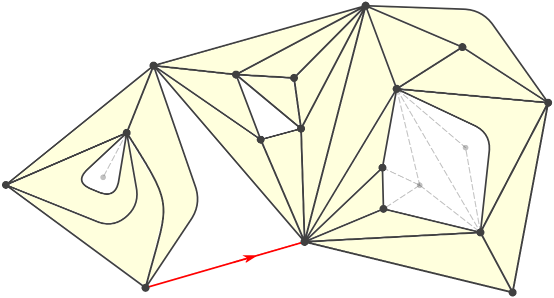


Hole perimeters:

$$\boldsymbol{X}^{(n)}(30) = (4, 3, 3, 2, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

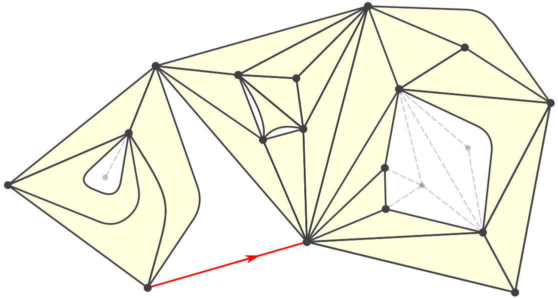


Hole perimeters:

$$\boldsymbol{X}^{(n)}(31) = (4, 3, 3, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

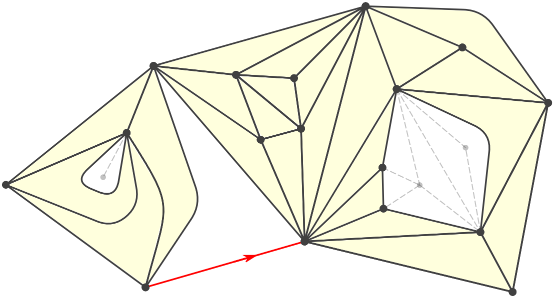


Hole perimeters:

$$\boldsymbol{X}^{(n)}(32) = (4, 3, 2, 2, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

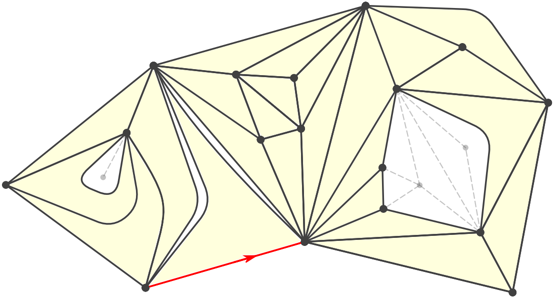


Hole perimeters:

$$\boldsymbol{X}^{(n)}(34) = (4, 3, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

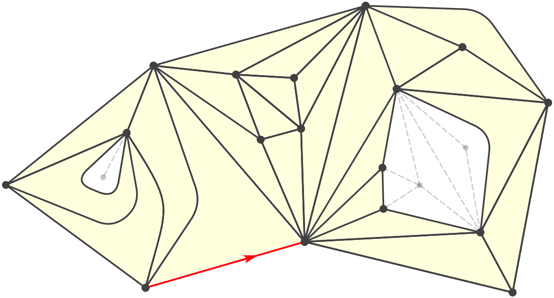


Hole perimeters:

$$\boldsymbol{X}^{(n)}(35) = (4, 2, 2, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

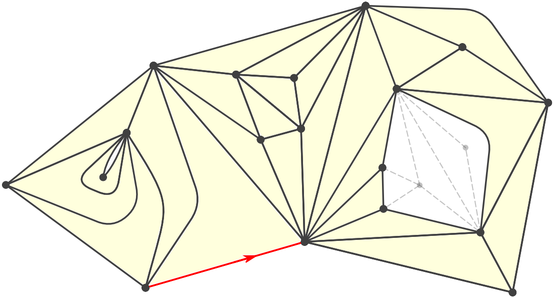


Hole perimeters:

$$\boldsymbol{X}^{(n)}(37) = (4, 1, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

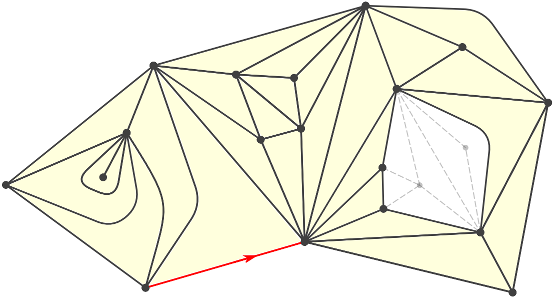


Hole perimeters:

$$\boldsymbol{X}^{(n)}(38) = (4, 2, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

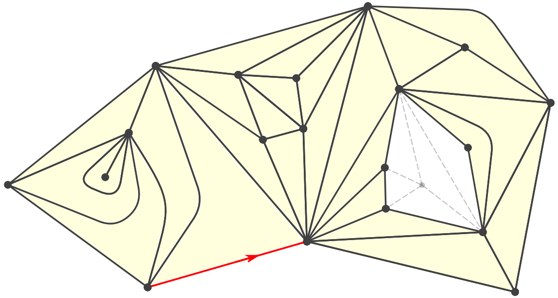


Hole perimeters:

$$\boldsymbol{X}^{(n)}(39) = (4, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

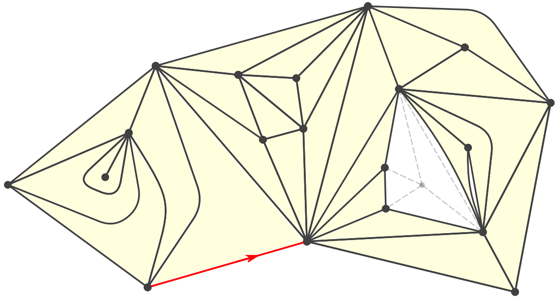


Hole perimeters:

$$\boldsymbol{X}^{(n)}(40) = (5, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

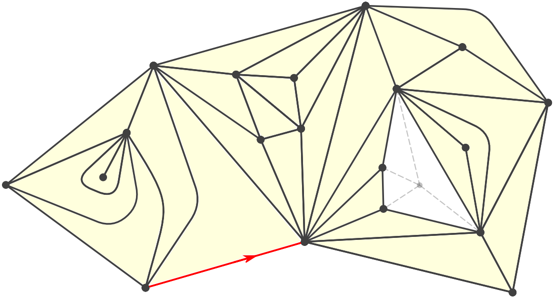


Hole perimeters:

$$\boldsymbol{X}^{(n)}(41) = (4, 2, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

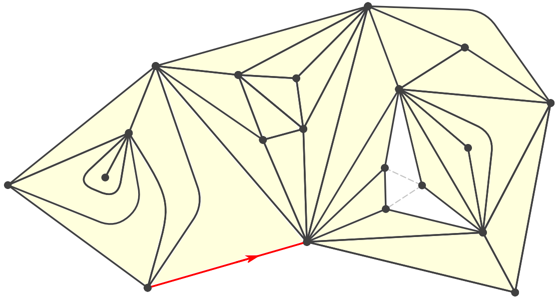


Hole perimeters:

$$\boldsymbol{X}^{(n)}(42) = (4, 0, \dots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

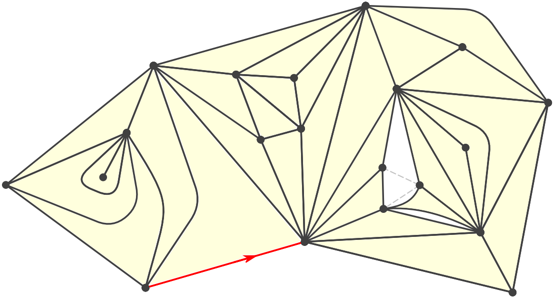


Hole perimeters:

$$\boldsymbol{X}^{(n)}(43) = (5, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

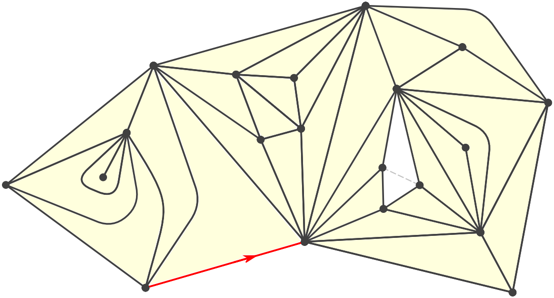


Hole perimeters:

$$\boldsymbol{X}^{(n)}(44) = (4, 2, 0, \dots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

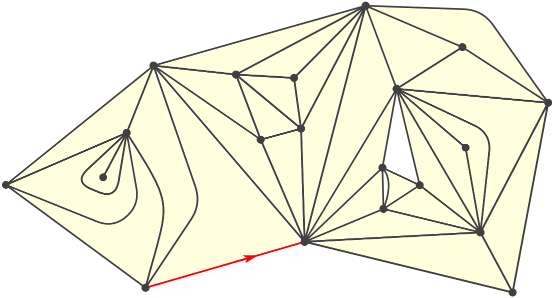


Hole perimeters:

$$\boldsymbol{X}^{(n)}(45) = (4, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon

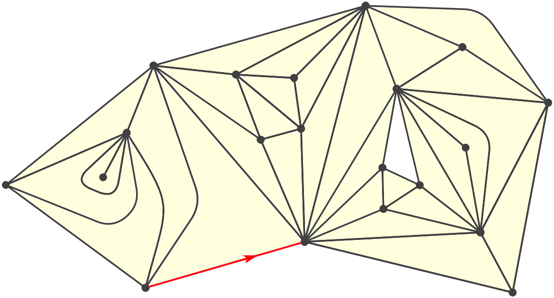


Hole perimeters:

$$\boldsymbol{X}^{(n)}(46) = (3, 2, 0, \ldots)$$

Motivating example

Peeling a random Boltzmann triangulation of the n-gon



Hole perimeters:

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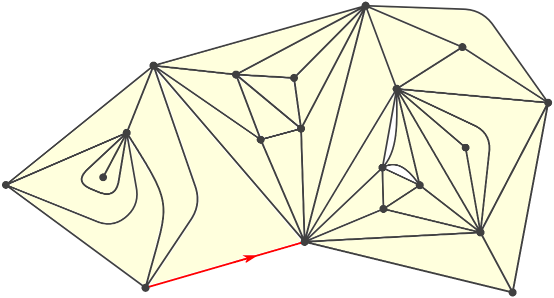
Motivating example

Peeling a random Boltzmann triangulation of the n-gon



Hole perimeters:

$$\boldsymbol{X}^{(n)}(48) = (2, 2, 0, \ldots)$$

Motivating example

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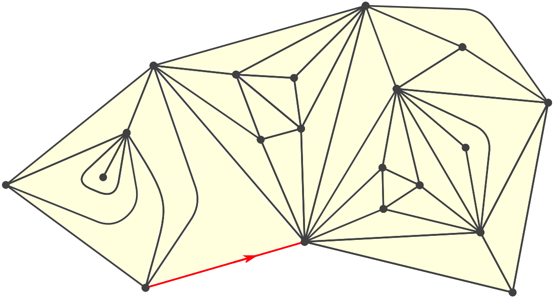

Hole perimeters:

$$\boldsymbol{X}^{(n)}(50) = (0, \ldots)$$

Motivating example

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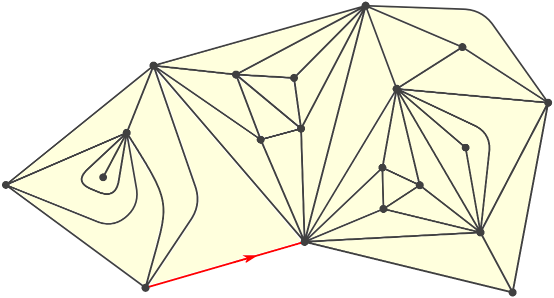
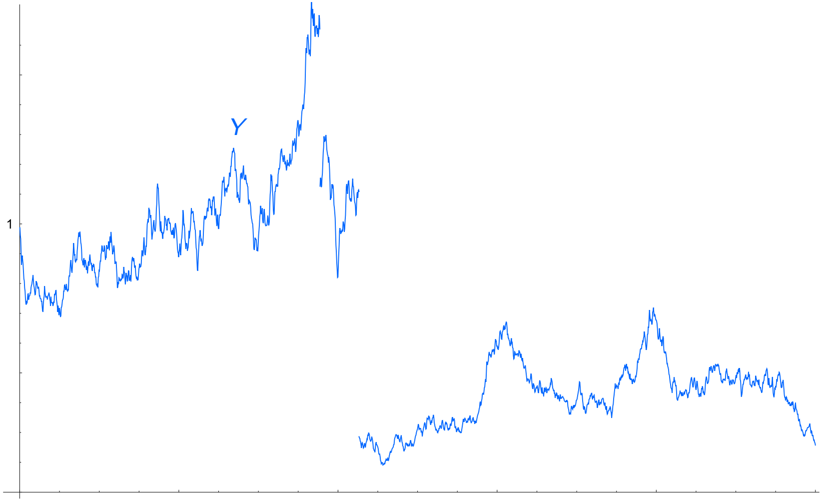


Fact (Bertoin et al., 2016, 2017). There is a scaling limit:

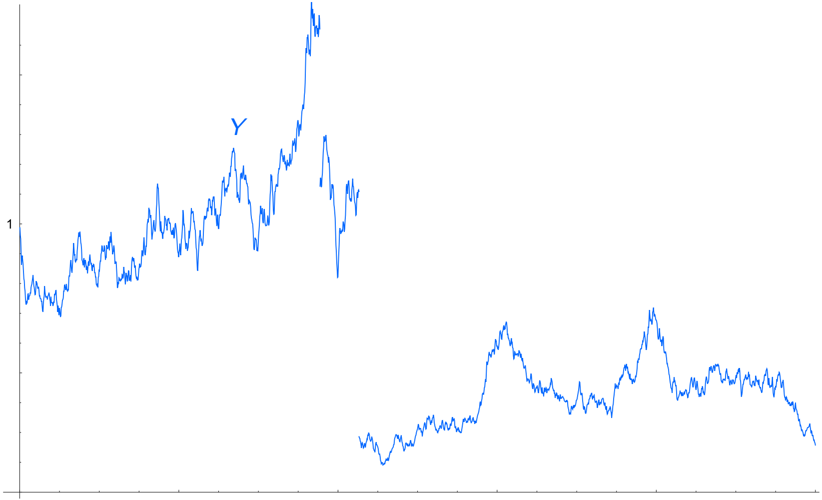
$$“ \quad \left(\frac{\widetilde{\boldsymbol{X}}^{(n)}(\lfloor a_n t \rfloor)}{n} : t \geq 0 \right) \xrightarrow[n \to \infty]{(d)} \boldsymbol{Y} \quad ”$$

Self-similar growth-fragmentation



Y is a positive, self-similar, Markov process

Self-similar growth-fragmentation



self-similar: law of Y under P_y = law of $yY\left(y^{-\gamma}\cdot\right)$ under P_1

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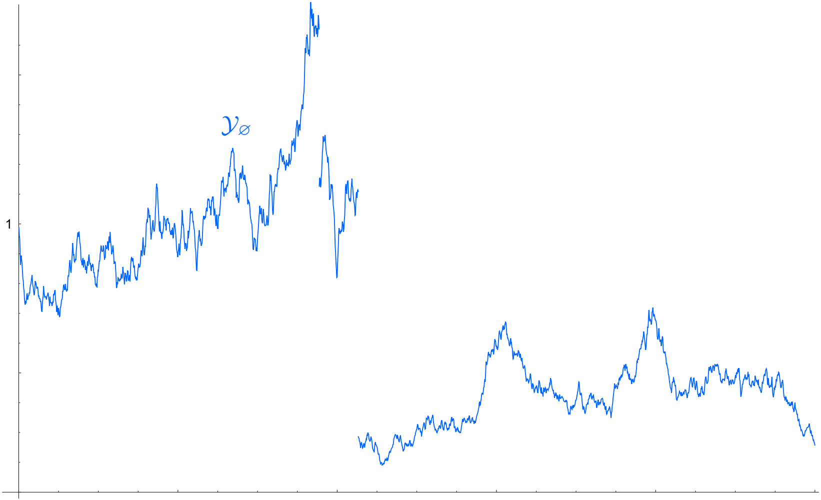

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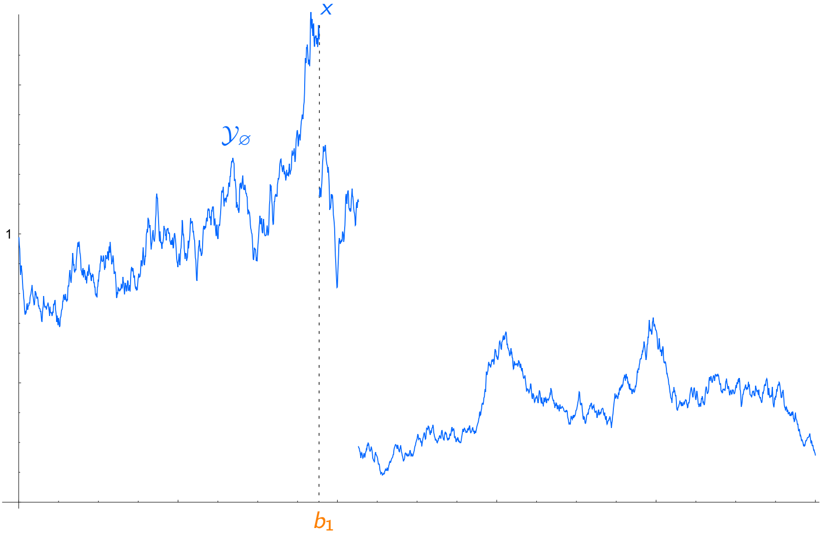

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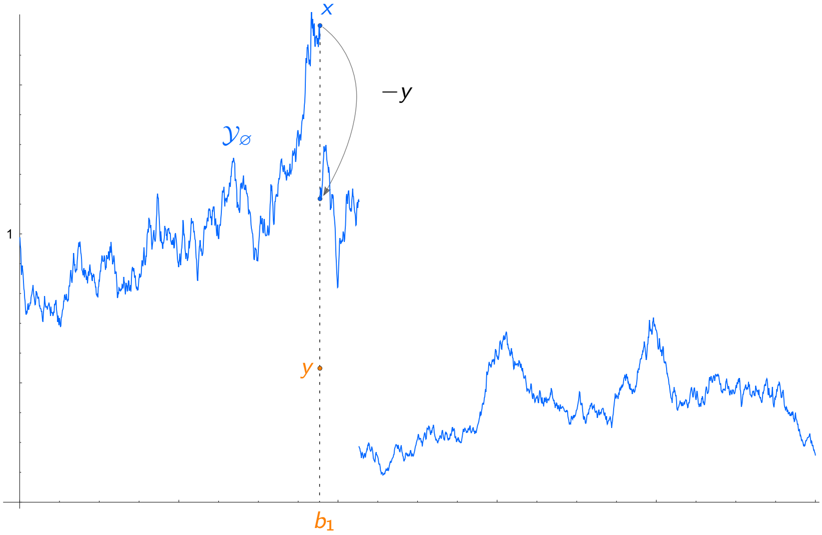

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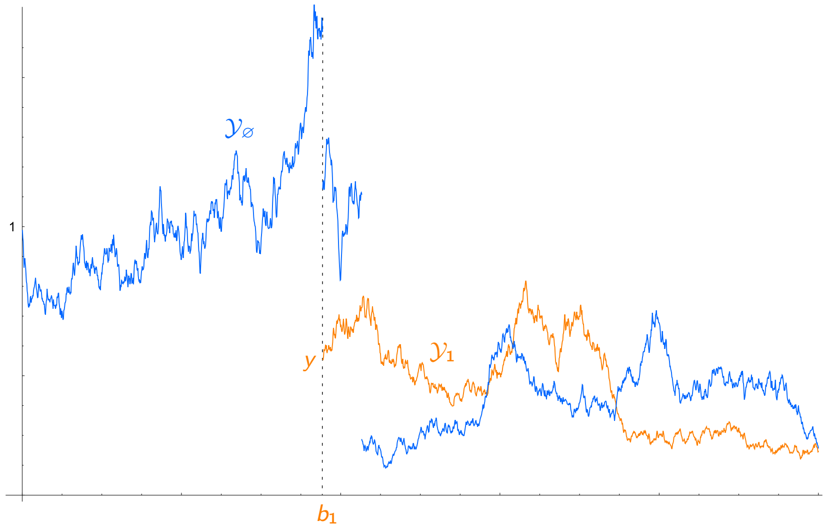

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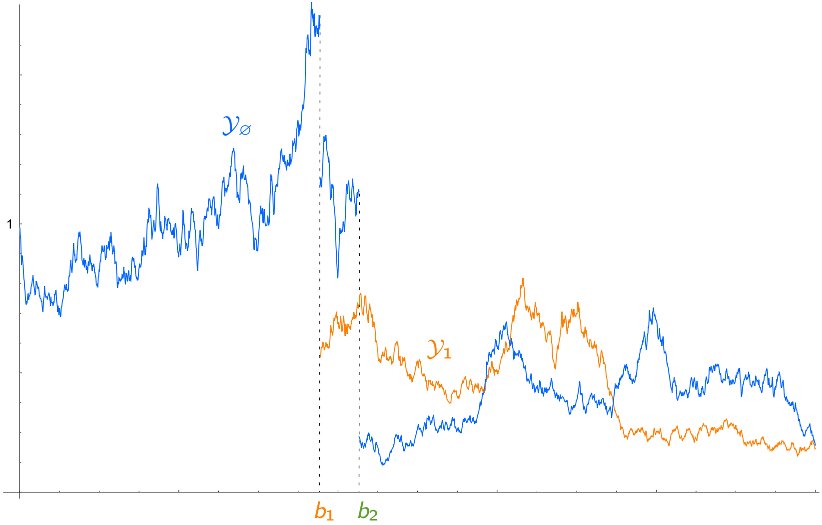

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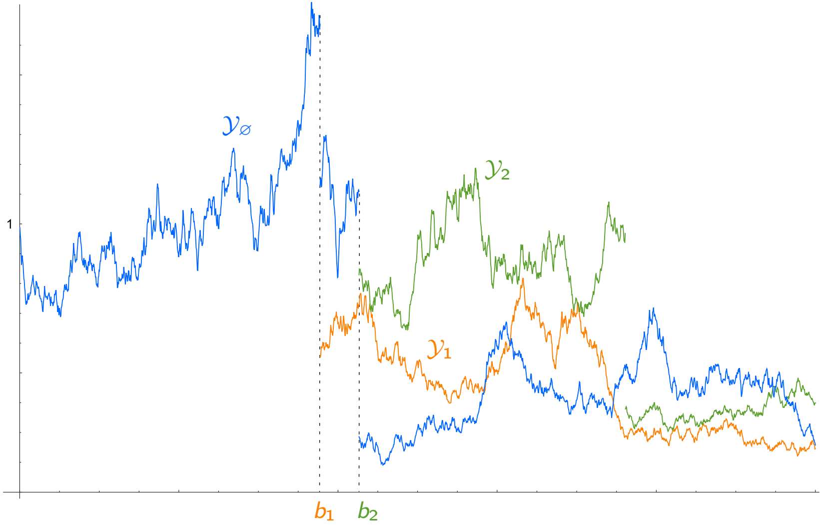

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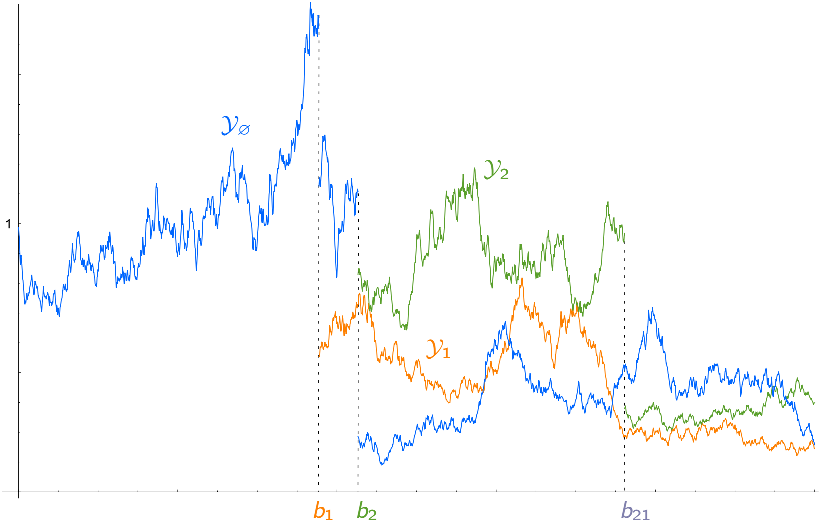

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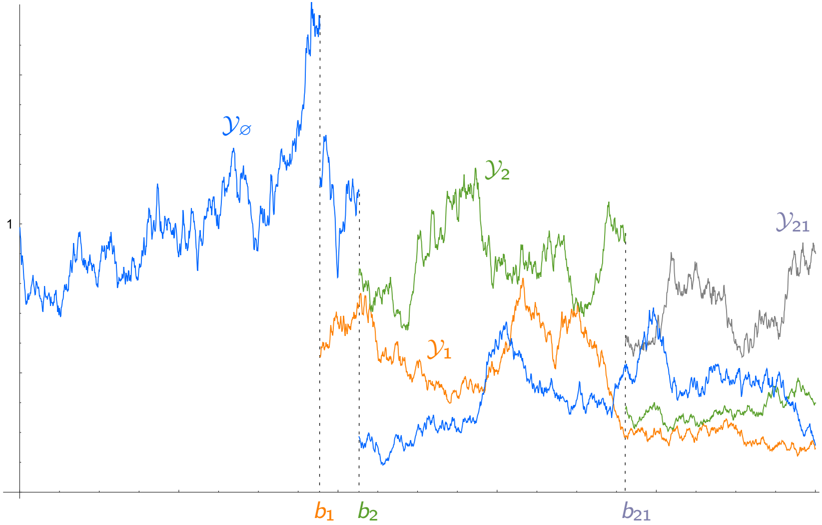

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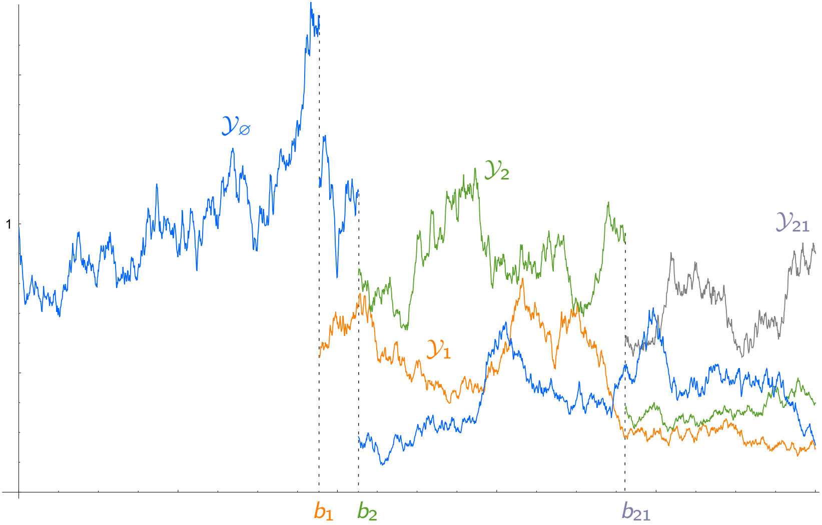
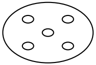


$$\boldsymbol{Y}(t) = \{\!\{\mathcal{Y}_u(t - b_u) \colon b_u \le t\}\!\} = \big(Y_1(t) \ge Y_2(t) \ge \cdots\big)$$

Markov branching process

$\boldsymbol{X}^{(5)}$:



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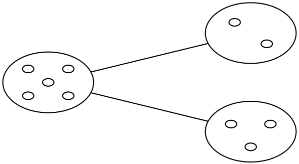

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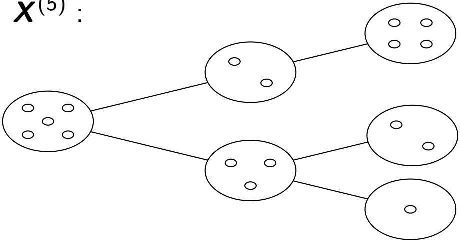

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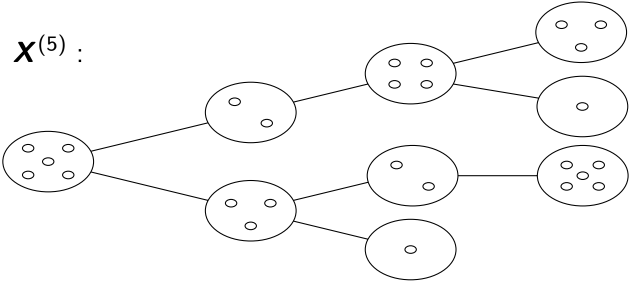

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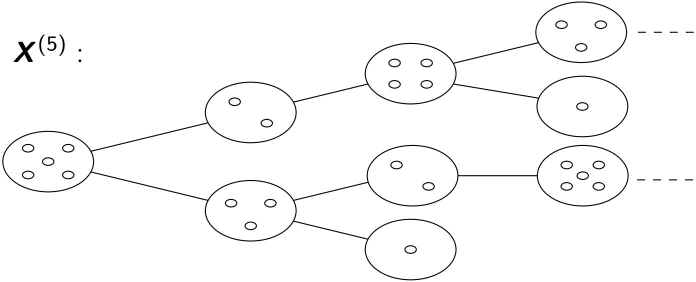

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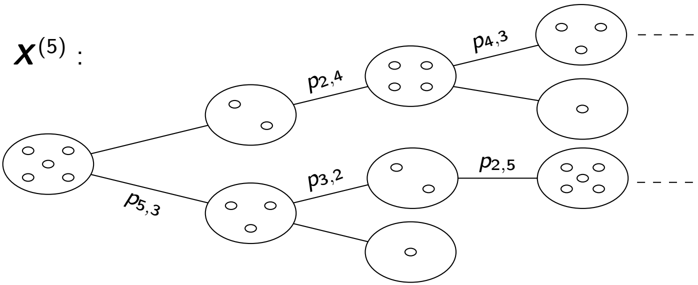

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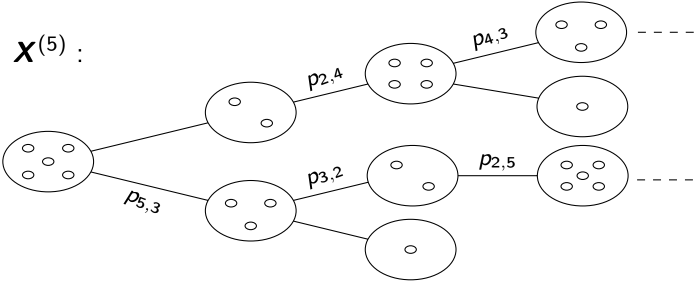

The system is entirely described through the Markov transition kernel $p_{n,k}$, $n \leq 2k$, of the *locally largest particle* $X^{(n)}$.

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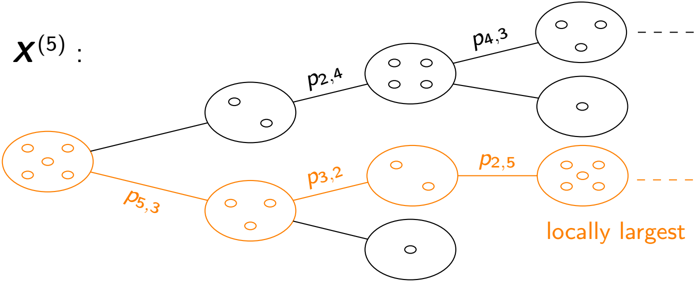

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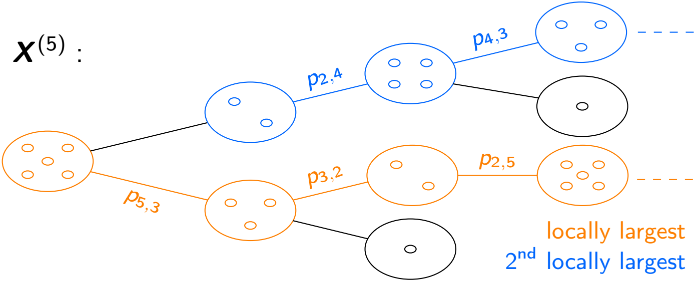

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Scaling limits

Which asymptotic conditions on $(p_{n,k})$ imply a scaling limit

1. for the Markov branching process $\boldsymbol{X}^{(n)}$?

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Starting point: scaling limit for the locally largest particle $X^{(n)}$.

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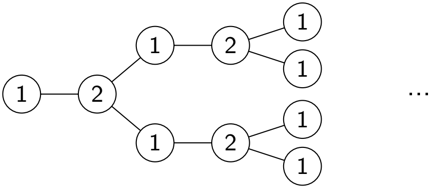

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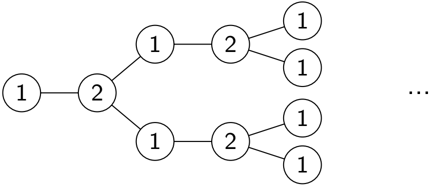

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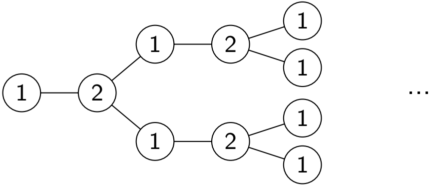

Since we do not (want to) deal with the behaviour of $p_{n,k}$ for "small" n, we must **prune** the system:

*Particles with size $\leq M$ (large but fixed) are **frozen**.*

$\Longrightarrow$ Locally largest particle stopped below M ($p_{n,n} := 1$, $n \leq M$).

Outline

Introduction

1 Motivating example

2 Self-similar growth-fragmentation

3 Markov branching process

4 Scaling limits

5 Two difficulties induced by growth

Results

6 Assumptions

7 Scaling limit for the process

8 Scaling limit for the tree

9-10 Proof aspects

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(H1) for all $t \in \mathbb{R}$,

$$\Psi_n(\mathrm{i}t) := a_n \sum_{m=1}^{\infty} p_{n,m}\left(\left(\frac{m}{n}\right)^{\mathrm{i}t} - 1\right) \xrightarrow[n\to\infty]{} \log \mathbb{E}[e^{it\xi_1}] =: \Psi(\mathrm{i}t);$$

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(H3) $\kappa(q^*) < 0$, and for some $\varepsilon > 0$,

$$\lim_{n\to\infty} a_n \sum_{m=1}^{n-1} p_{n,m}\left(1 - \frac{m}{n}\right)^{q^*-\varepsilon} = \int_{\mathbb{R}_-} \left(1 - e^y\right)^{q^*-\varepsilon} \Lambda(\mathrm{d}y),$$

with Λ Lévy measure of ξ, $\kappa(q) := \Psi(q) + \int_{\mathbb{R}_-} (1 - e^y)^q \, \Lambda(\mathrm{d}y)$.

Scaling limit for the process

Theorem 1. Under (H1)–(H3), we can fix M large so that

$$\left(\frac{\boldsymbol{X}^{(n)}(\lfloor a_n t \rfloor)}{n} : t \geq 0\right) \xrightarrow[n\to\infty]{(d)} \boldsymbol{Y}$$

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The limit $\boldsymbol{Y}$ is the self-similar growth-fragmentation driven by Y, where

$$\log Y(t) = \xi\left(\int_0^t Y(s)^{-\gamma}\,\mathrm{d}s\right), \qquad t \geq 0.$$

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Theorem 2. Under (H1)–(H3), $q^* > \gamma$, we can fix M so that

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The limit $\mathcal{Y}$ is the continuum random tree associated with $\boldsymbol{Y}$, as constructed by Rembardt and Winkel (2016).

Proof aspects

Convergence of "finite-dimensional marginals"

- (H1)-(H2) provide the scaling limit for $X^{(n)}$ and its absorption time, thanks to a criterion of Bertoin and Kortchemski (2016).

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- (H3) adds the convergence of the daughter sizes at birth.
- Convergence of finite subfamilies of $\boldsymbol{X}^{(n)}$ and subtrees of $\mathcal{X}^{(n)}$ (by induction).

Proof aspects

Tightness

- Let $\kappa_n(q) := \Psi_n(q) + \sum_{m=1}^{n-1} p_{n,m}\left(1 - \frac{m}{n}\right)^q$.

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- Since $\kappa(q^*) < 0$ by (H3), we have a supermartingale

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- Uniform control of $\mathtt{height}(\mathcal{X}^{(n)})$ by Foster–Lyapunov means.

Thank you!